WHEN A PLAYER TELLS YOU YOU'VE GOT THE ODDS OF THE GAME WRONG, READ THIS:

THERE IS ONLY ONE KIND OF QUESTION TO BE ASKED AND ANSWERED IN THIS EXPLANATION: "HOW MANY WAYS ARE THERE TO DO THAT?"

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Every now and then, we get a challenge from a player about the odds we have posted for a draw game. In my lottery, these math-sounding questions come to me. I look on these occasions as a chance to try to understand where the player is coming from, and lead them from there to a view that lines up with the actual properties of the game. In other words, I try to teach something about probability. I have found that players generally appreciate this, and end up feeling better about the lottery.

Currently, lots of people seem to regard things they "find on the Web" as authoritative. If you search the Web with terms like "lottery game odds," you may find some calculators that do a good job of calculating the odds of winning the top prize of a lotto-type game. An example is http://www.webmath.com/lottery.html. This site has an interactive calculator, and includes a basic discussion of the reasoning that drives the calculation. A similar discussion, without a calculator, can be found at http://www.ehow.com/info_8490895_odds-winning-lottery.html. Still another choice, less helpful to a player because it "tells" with authority, rather than "explaining" is http://statistics.about.com/od/Applications/a/What-Are-The-Odds-Of-Winning-The-Lottery.htm.

None of the sources mentioned above addresses anything beyond the odds of winning the top prize in a game - that is, "matching all the balls." I have found that questions from players more often challenge the odds of the lower-tier prizes – "matching some of the balls." Sometimes, players apply what they have learned about "matching all" to the more complex situation of "matching some," and miss some important piece. That is when they write to tell us that our odds are wrong and need to be changed.

Probably the objection I have heard most often runs like this:

"There are 15 balls in the Mega Ball tumbler. Clearly, your odds of picking the winning Mega Ball are 1 in 15! And yet you say that the odds are 1 in 21.4! Why can't you get that right??"

My response is that we don't play just the Mega Ball alone, but the other five numbers as well, and that we need to think about the likelihood of picking five numbers from 75 and getting not a single match; because (and here's the "bright side") matching the Mega Ball and failing to match anything else is less likely than just matching the Mega Ball. The difference between 1 in 15 and 1 in 21.4 has to do with the likelihood of winning prizes bigger than break-even.

Optimistic people like it when they understand that if the odds of winning just a breakeven prize are not as good as they thought, it is because of the odds of something better happening.

Players generally don't insist on knowing how we get from 1 in 15 to 1 in 21.4 precisely. But sometimes they do, and sometimes players ask questions about other winning combinations. If the lottery cannot explain how it arrives at the published odds, it risks seeming less than credible.

Wikipedia, as usual, has a good and thorough explanation for those who are interested, at https://en.wikipedia.org/wiki/Lottery_mathematics. However the math notation in that article is a barrier for many potential readers. For this reason it is generally not a good resource for lottery players. The Wikipedia article does, very helpfully, point out some functions found in Microsoft Excel (and similar applications) that are useful in making calculations.

My purpose in this note is to give a plain-language explanation, suitable for a lottery player, of how to understand the odds of any prize tier in a lotto-type game. By a lotto-type game, I mean a game in which a set number of objects (balls, symbols, numbers, letters, whatever) are selected without replacement from a field of a certain size, in the same way by both the player and the lottery. The win in a lotto game depends on matching some or all of the objects drawn, and not upon the order of drawing.

Other kinds of games, such as the classic "Pick 3" numbers game and Keno, can be understood as variations on the basic lotto concepts.

The lottery is going to draw only one set of numbers; for clarity we will call these the "drawn numbers." The player has a separate bet on each set of numbers she picks, and we will be talking about the odds of any one set of her picked numbers matching the lottery's set.

There is only one kind of question to be asked and answered in this explanation: "How many ways are there to do that?" We ask this question at several points.

Let's take as an example the classic "six of 49" lotto game offered by several jurisdictions. The player and the lottery are going to select six numbers from the range 1 through 49. Since the player already has a ticket, let's talk about the likelihood that the lottery "draw" matches the all numbers on the player's ticket – the player's "pick." This is the top-prize win. The fundamental question is, "How many ways are there to pick six numbers from the field one through 49?" The player has one way; the lottery has one way; the likelihood that they match is one out of the total number of combinations.

At the start there are how many ways that the lottery's draw can match the players pick? The lottery 's first number drawn could match any one of the six numbers in the pick. How many ways can the lottery draw? There are 49 possibilities.

Let's represent the likelihood of a match as the number of ways that the lottery can draw, divided by the number of ways the number drawn could match the player's pick: 49/6. In odds notation, the likelihood that the first number drawn by the lottery will match one of the six chosen by the player is 1 in (49/6), or 1 in 8.333.

Now supposing that the first number was a match, how many ways can the lottery's second drawn number match the player's pick? Only five numbers remain unmatched in the pick. The lottery has 48 possibilities remaining. So the likelihood of matching the second number is one in 48/5, or 1 in 9.667

This pattern repeats until all the possibilities of

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matching have been exhausted. Each successive match is less likely, since the number of ways to match decreases quicker than the number of ways to draw. The likelihood of matching all six numbers is 1 out of the whole space of (49/6*48/5*47/4*46/3*45/2*44/1) combinations, or 1 in 13,983,816, or about 1 in 14 million. This is the number of distinct combinations that can be made taking 49 objects, six at a time.

There are a number of resources on the Web that will confirm this result, without necessarily guiding their users to understand why. However, by extending the kind of reasoning used above, we can get to an understanding of the subtler question of partial matches.

How about the likelihood to match 5 of the 6 numbers, to win the second prize? Here, we need to think not only of the ways the lottery draw could match the player's pick, but also the number of ways it could NOT match. Note that when the lottery is done drawing, there are still 43 numbers left. At any point in the action, there are 43 ways that the lottery could draw and avoid matching any of the player's numbers.

How many different ways can the lottery "miss" exactly one of the player's six numbers? Clearly, six ways. How many ways are there to substitute a different number? Forty-three. Taking these two facts together, there are (6 * 43=) 258 ways the lottery draw could miss the player's pick by just one number. The likelihood of doing this (and winning the second prize) is thus 258 times greater than winning the top prize itself. In other words, the likelihood of winning the second prize is 1 in 13 million something (13,983,816, exactly) divided by 258, or about 1 in 54,200.

Most lotto games have lower prize tiers as well. These are easier to win because there are more ways to miss two or three numbers than there are to miss none, or only one. In fact, you can verify by listing them that there are 15 ways to miss two out of the six numbers in the player's pick (the first and second, the first and third, the first and fourth, and so on.) There are 43 leftover numbers available for the first, and 42 for the second. Since the order in which these are chosen (of the two orders possible) does not matter, we say that we have (43*42/2) = 903 different combinations to pick from and 15 ways to use them, in order to miss exactly two out of the six numbers. Consequently there are (903*15 =) 13,545 combinations out of the 13.99 million total that would match exactly four of the player's numbers. We usually state this as "the odds of matching four out of six are 1 in 1,032."

Missing exactly three numbers is even easier. There are 20 ways to miss three numbers in the pick (first, second, and third; first, third, and fourth, and so on). There are six possible orders in which to arrange three objects, so the operation of choosing from 43, then 42, then 41 numbers gives (43*42*41/6 =) 12,341 distinct combinations. Consequently there are 20* 12,341 = 246,820 ways the lottery can match exactly three numbers out of the player's pick: this gives odds of 1 in 56.67

The classic 6-of-49 lotto game does not pay tickets that match one or two numbers – these matches are too abundant, and in fact they (along with the "no match" tickets) are the source of the money that pays the prize winners. Applying the same logic as above, you can verify that there are:

(43*42*41*40/15 =) 1,851,150 ways to miss four numbers,

(43*42*41*40*39/6 =) 5,775,588 ways to miss five, and

(43*42*41*40*39*38/1 =) 6,096,454 ways to miss them all.

Adding these together, we see that the non-winning combinations comprise more than 98 percent of the possible combinations. The fortunate 1.864 percent of wagers win money. Put another way, the odds of winning any prize with a single pick in lotto are 1 in (1/.01864 =) 53.65.

To summarize: the classic lotto-type lottery game can be understood by considering all the ways a player's pick can match, or not match, the lottery's drawn numbers. I have presented a way of calculating the odds of each outcome exactly, using only basic arithmetic. I hope it may be useful when players are curious enough to challenge us to explain our published odds! ■

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Note:

[1] "verify by listing 15 ways:" sure, there's a formula for this, but again it's apt to be off-putting since it involves factorials. We write it like this: combinations of 6 things taken two at a time = (6!/(6-2)!*2!), where 6! is read "six factorial" rather than "six bang!"