

SIMULATED PLAY, REAL GAINS?

We may, by listening to potential players, hear some ideas that sound like something other than Numbers, Lotto or Keno. This would be refreshing. Rather than rejecting these as 'not lottery games,' we can try to codify them, with suitably random elements, and let large-scale simulations show us what they can do.

STEPHEN WADE, PRINCIPAL, LOTTERY MANAGEMENT CONSULTING, LLC.



Over the Christmas holiday, I got to visit with my daughter, who was between semesters in school.

As we have done for many years, we brought out the Scrabble game and she won a game or two. I like Scrabble. It is about words, and it insists on putting letters together in spatial order, to mean something. Putting words together to make meaning is natural to us, as speech. Putting letters together perhaps draws on similar resources as speech, but it exposes a lack of fluency in all of us. The letters, after all, have no meanings in themselves; one thing leads to another only as sounds. Those who like the game find this amusing, engaging and maybe even tiring. One game of 500 points tends to be enough.

"Let's play something different," she said. "How about those lottery games you work on? How do you even play those?"

In our household, as in most households of people who work for lotteries, no one plays lottery games. The idea of a game where you can win money is appealing enough, but how to go about it is not obvious.

Sticking to the basics, I said, "You pick some numbers, the lottery picks some numbers; if they match, you win."

"Why does it have to be numbers?"

"It doesn't. It's just about matching. We could play with letters. We could play with

these tiles. Take the simplest game, maybe Pick 3. Let's take 10 letters – maybe the first 10? And I'll show you."

We sorted through the tiles, whereby I was reminded that 'F' and 'J' are very rare in Scrabble, but at length we got the set: ABCDEFGHIJ. I turned them face down and scrambled them on the table. "OK now you're the player, take one, write it down, put it back, and scramble. Do that three times."

She did so.

"Now I'll be the lottery; I do the same thing. You lose. Odds are 1 in 1,000 to match exactly; better odds to match in part. Did I mention that most people don't play this for amusement?"

"What if we didn't put the tiles back?"

"Then it would be like Lotto, a different game."

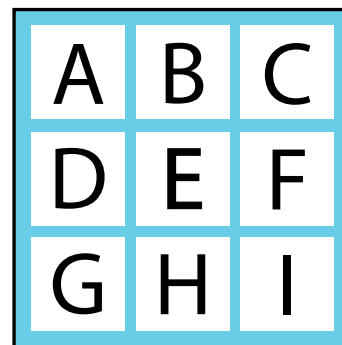
She said, "What if we drew them all, so it's just about order?" She sorted the 10 tiles in order and pushed them into a string.

"That would be different still. I don't know of a lottery game like that. You could do that. If there's only one right order, how hard it is depends on how many symbols you use. That could get pretty hard, pretty quick. Let me see."

In my house, I am never far from a way to compute N factorial, so I came back with: "For 10 symbols it would be about 3.6 million to one; with nine, about 363,000 to one."

"That sounds too hard to be much fun. Couldn't we do something more interesting? What if you pushed them together and scored on any way to get them touching, in order?" She pushed the pieces from the string together into a loose patch. "Nine fits better than 10." She put the 'J' aside and made a square of Scrabble tiles, 3x3.

"Best score is all touching an alphabetical order. So, the best you could get would be:"



"But," I pointed out by moving tiles, "this would be just as good:

C	B	A
D	I	H
E	F	G

"And if diagonals count, you could do:

A	B	C
E	D	I
F	G	H

"So, it looks like there may be several ways of arranging that are all equally good. That may not work for a lottery game. You only want one winner."

"Why is that?" she asked. "That seems like a very twentieth-century attitude."

"Ok, so plays that are equally good could share a prize, but we'll have to know how many are equally good. And if the top prize needs all nine letters touching, what do you get if it's close but interrupted?"

"Nine letters touching means eight points of contact – call them links? So you could just count links, and if there are seven you could pay a smaller prize..."

"How big a prize? We would need to know how likely it is to get eight links, or seven, and so on. And this looks simple but it's really kind of complicated. The places aren't all the same: The corners have three neighbors, the sides have five and the center has eight! So, you're always going to get a link from the center."

And that was about the limit of my theoretical insight. But the idea of a lottery game that had some geometry

to it appealed to me, and the physical action of moving the tiles around with their faces concealed and then turning them up felt like something that would translate from the Scrabble board to my phone. So I wanted to understand how this might work as a lottery game. Pretty quickly I realized that the theory was not like any lottery game I knew of, even though I understood perfectly how to play the game. This was frustrating until **I realized that I understood enough to teach a computer how to play the game.** So rather than worry about the theory, I spent a couple of enjoyable hours building an Excel workbook to play the game. I laid out a wager on a grid and worked out how to score it by counting links.

I will sketch what I did, not to try to impress you with my Excel skills, but to reveal that it was really pretty simple. A key was to consider the points of contact between cells of the 3x3 grid: There are 20, in fact (of which eight join the center cell to its neighbors.) If the symbols in these cells are adjacent in alphabetical order, that point of contact has a link. Since Excel is good at alphanumeric sorting, it is easy to convert a sequence of letters (GDHEACFBI, in left-to-right, top-to-bottom reading order) to a sequence of numbers. Then, when the numbers in adjacent cells differ by one, this identifies a link.

G	D	H
E	A	C
F	B	I

7	4	8
5	1	3
6	2	9

Starting with the center cell, for instance, we can evaluate links by putting, in each of the eight neighbor cells, an IF statement that returns "1" if the contents are in sequence (i.e. occupant of center cell is 1 greater or 1 less than the occupant of the neighbor cell, and "0" otherwise. Each of the corner cells has another two points of contact to evaluate, and the side cells have among them four diagonal contacts, for a total of 20 questions to be asked. The score for the play is then just the total of links: five in the example above.

7	4	8
5	1	3
6	2	9

Of course, a human might play visually, without thinking any of this. My point here is that we don't have to mimic the way a human plays the game; we just need be able to score it correctly. Once I was satisfied with this part, I wrote some code to run in the background to do this over and over, recording the wager and writing the results each time. I verified the result manually ten times, then let it run 100 times and verified the last 10, then 1,000 times to get an estimate of how long it would take to get one million results. Finally, I set up the index for one million and let it run – for hours, as it turned out, on my machine.

The result that I got from the first million suggested the sort of prize table that is shown on the next page.

What I thought was striking about it was the abundance of middling results; that is, four or five links out of the possible eight. These might correspond to near-wins – "close, but just out of the money." This is a key feature of Keno, and in my view the main reason Keno works so well for repeated play. I pointed this out to my daughter, who supposed I could be forgiven for thinking that way.

"Now can you put some Scrabble back into this? How about prizes for making words?"

Simulated Play Table

Links	Games	Apparent Odds 1 in	Prize	Prize Cost for \$1 bet
8	2,089	479	\$75.00	\$0.157
7	30,528	32.8	\$6.00	\$0.183
6	147,855	6.76	\$2.00	\$0.296
5	301,882	3.31	\$0.00	-
4	311,405	3.21	\$0.00	-
3	162,767	6.14	\$0.00	-
2	40,219	24.9	\$0.00	-
1	3,255	307	\$0.00	-
Total	1,000,000			\$0.635

I balked at this, but then realized that the list of real English words that could be made this way would be short, and that my patient computer could tabulate how often they occurred. So even randomly spelling words could be a feature of a lottery game. How about paying the top prize only if “BIG” appears in the top row of a random scramble? I certainly have the means to evaluate that.

The point of this article is not that the little game described here has useful properties (even though I think it does). The point here is that it is not necessary to start from math that we completely understand to build lottery games that are entertaining for a human. Rather, we can start from some human’s idea of what might make an entertaining game, then express the rules so that we can discover the game’s properties by letting a computer play it. Computers are patient: You can play as long as it takes to get precision. Several runs of one million games gave slightly different counts for each outcome, but over the long run the estimate for eight links in this game stabilized at 1 in 463.

The simulation method is my message. And the method may be surprisingly accessible. The example described here did not depend on arcane knowledge, or a specialized computing platform. Rather, we used the ubiquitous Excel platform and familiar functions. This is not to claim that Excel is absolutely the best platform for this sort of work, but rather that it is plenty good enough to get useful results.

THIS IS AN INVITATION TO LOOK AT THE PLAY AND THE PLAYERS FIRST, AND THE PROBABILITIES LATER.

This is an invitation to look at the play and the players first, and the probabilities later. Not every concept that looks interesting to a player will support a lottery game prize structure that makes sense. Simulations may expose properties that need “fixing,” or they may reveal that the concept is simply unworkable, from a quantitative point of view. Lotteries may have presumed that new game ideas will come from the industry’s vendors, through some process inaccessible to the lotteries themselves. My assertion is that we may, by listening to potential players, hear some ideas that sound like something other than Numbers, Lotto or Keno. This would be refreshing. Rather than rejecting these as “not lottery games,” we can try to codify them, with suitably random elements, and let large-scale simulations show us what they can do. The tools are within your reach. And we know that picking up tools and making things can be very empowering. A lottery that takes this up is likely to feel a win through engaging the skills of lottery staff, even if it does not immediately result in a new lottery game. Why not play? ■

Stephen Wade, Principal, Lottery Management Consulting, LLC.